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PRODUCTION INVENTORY SYSTEM WITH WEIBULL DETERIORATION FUNCTION

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ABSTRACT:
In financial parlance, inventory is do fined as the sum of the value of raw materials, fuels and lubricants spare parts, maintenance consumable some processed material and finished goods stock at any given point of time. The operational definition of inventory would be the amount of raw materials fuel and lubricants. Spare parts and semi-processed material to be stocked for the smooth running of the plant. Inventories are maintained basically for the operational smoothness. They can effect by uncoupling successive stages of production, where as the monetary value of inventory serves as a guide to indicate the size of the investment made to achieve this operational convenience. The materials management department is expected to provide this operational convenience with a minimum possible investment in inventories. The objectives of inventory operational and financial needless to say are conflicting the materials department is accused of both stock outs as well as large investment in inventories. The solution lies in exercising a selective inventory central and application of inventory central techniques.
Key words : Inventory, financial needless

INTRODUCTION:

In this paper an attempt has been made to develop an inventory model for infinite planning horizon with exponentially increasing demand rate. It can be noticed that deterioration does not depends upon time only. It can affect due to whether conditions, humidity, storage conditions etc. therefore it is more realistic to consider deterioration rate as two parameter weibful distribution function. Shortage are allowed and fully backlogged. The holding cost considered a linear function of time. The optimal solution of t he proposed inventory model is derived and considered same cases.

ASSUMPTIONS AND NOTATIONS:

- The replenishment size is constant and production is instantaneous during prescribed time period T of each cycle.
- Lead time is zero.
- Shortage are permitted and completely accumulated
- Demand rate $D(t)=\dfrac{d}{(e-1)T}e^{t/T}$ at any time t.
- Deterioration rate $\theta=\alpha\beta t^{\beta-1},\ 0<\alpha<1,\ \beta\geq 1$
- Holding cost $C_1=h+\gamma t$ per unit.
- $C_1,\ C_2$ are the cost of each item, shortage cost per unit per unit time respectively.

MATHEMATICAL MODELING AND ANALYSIS FOR THE SYSTEM:

Let I(t) be the current stock level at any time t.

$$\frac{dI(t)}{dt}+\theta I(t)=-\frac{d}{(e-1)T}e^{\frac{t}{T}}, \qquad 0\le t\le t_1 \qquad \ldots(1.1)$$

$$\frac{dI(t)}{dt}=-\frac{d}{(e-1)T}e^{\frac{t}{T}}, \qquad t_1\le t\le T \qquad \ldots(1.2)$$

By equation (1.1), we have

$$\frac{dI(t)}{dt}+\alpha\beta t^{\beta-1}\cdot I(t)=-\frac{d}{(e-1)}e^{\frac{t}{T}}$$

$$\text{I.F.}=e^{\int \alpha\beta t^{\beta-1}dt}=e^{\alpha t^{\beta}}.$$

Solution of equation (1.1) is given by

$$I(t)e^{\alpha t^{\beta}}=-\int \frac{d}{(e-1)T}e^{\frac{t}{T}}e^{\alpha t^{\beta}}dt+B$$

where B is the constant of integration.

$$=-\frac{d}{(e-1)T}\int e^{\frac{t}{T}}\left(1+\alpha t^{\beta}\right)dt+B.$$

After expanding $e^{\frac{t}{T}}$ by Taylor's series and neglecting higher order terms of $\frac{t}{T}$ greater than 1 $\left(\because \frac{t}{T}<1\right)$, we get

$$I(t)e^{\alpha t^{\beta}}=-\frac{d}{(e-1)T}\int\left(1+\frac{t}{T}\right)\left(1+\alpha t^{\beta}\right)dt+B$$

$$=-\frac{d}{(e-1)T}\int\left(1+\frac{t}{T}+\alpha t^{\beta}+\frac{\alpha}{T}t^{\beta+1}\right)dt+B$$

$$=-\frac{d}{(e-1)T}\left[t+\frac{t^2}{2T}+\frac{\alpha t^{\beta+1}}{\beta+1}+\frac{\alpha t^{\beta+2}}{T(\beta+2)}\right]+B. \qquad \ldots(1.3)$$

At $t=0,\quad I(t)=S$, then from equation (1.3), we have

$$I(t)e^{\alpha t^{\beta}}=-\frac{d}{(e-1)T}\left[t+\frac{t^2}{2T}+\frac{\alpha t^{\beta+1}}{\beta+1}+\frac{\alpha t^{\beta+2}}{T(\beta+2)}\right]+S. \qquad \ldots(1.4)$$

At $t=t_1,\ I(t)=0$, then from equation (84), we have

$$S=\frac{d}{(e-1)T}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}\right]. \qquad \ldots(1.5)$$

Substituting the value of S in equation (1.4) from equation (1.5), then

$$I(t)=\frac{d}{(e-1)T}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}-t-\frac{t^2}{2T}-\frac{\alpha t^{\beta+1}}{\beta+1}-\frac{\alpha t^{\beta+2}}{T(\beta+2)}\right](1-\alpha t^{\beta}). \qquad \ldots(1.6)$$

Neglecting higher order terms of α , we get from equation (1.6)

$$I(t)=\frac{d}{(e-1)T}\left[t_1-t+\frac{t_1^2}{2T}-\frac{t^2}{2T}-\alpha t_1 t^{\beta}+\alpha t^{\beta+1}-\frac{\alpha t_1^2 t^{\beta}}{2T}+\frac{\alpha t^{\beta+2}}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}-\frac{\alpha t^{\beta+1}}{\beta+1}-\frac{\alpha t^{\beta+2}}{T(\beta+2)}\right]$$

$$I(t)=\frac{d}{(\ell-1)T}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}-t-\frac{t^2}{2T}-\alpha t_1 t^{\beta}-\frac{\alpha t_1^2 t^{\beta}}{2T}+\frac{\alpha \beta t^{\beta+1}}{\beta+1}+\frac{\alpha \beta t^{\beta+2}}{2T(\beta+2)}\right] \ldots(1.7)$$

Solution of equation (1.2) is given by

$$I(t)=\frac{d}{e-1}\left(e^{\frac{t_1}{T}}-e^{\frac{t}{T}}\right). \qquad \ldots(1.8)$$

Total amount of deteriorated units

$$D=S-\int_0^{t_1}\frac{d}{(e-1)T}e^{\frac{t}{T}}dt$$

$$=S-\frac{d}{(e-1)T}\left[Te^{\frac{t}{T}}\right]_0^{t_1}$$

$$=S-\frac{d}{(e-1)}\left(e^{\frac{t_1}{T}}-1\right) \qquad \ldots(1.9)$$

Substituting the value of S from equation (1.3) in equation (1.6), we get

$$D=\frac{d}{(e-1)T}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}-Te^{\frac{t_1}{T}}+T\right]. \ldots(1.10)$$

Number of units in shortage

$$=\int_{t_1}^{T}I(t)dt$$

$$=\frac{d}{e-1}\left[2Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}-eT\right].$$

Therefore average shortage cost

$$=\frac{C_2 d}{T(e-1)}\left[2Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}-eT\right] \qquad \ldots(1.11)$$

Average holding cost

$$=\frac{1}{T}\int_0^{t_1} I(t)(h+\gamma t).dt$$

$$=\frac{1}{T}\int_0^{t_1} hI(t)dt+\frac{1}{T}\int_0^{t_1}\gamma tI(t)dt$$

$$=\frac{h}{T}\frac{d}{(e-1)T}\int_0^{t_1}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}-t-\frac{t^2}{2T}\right.$$
$$\left.-\alpha t_1 t^{\beta}-\frac{\alpha t_1^2 t^{\beta}}{2T}+\frac{\alpha\beta t^{\beta+1}}{\beta+1}+\frac{\alpha\beta t^{\beta+2}}{2T(\beta+2)}\right]dt$$
$$+\frac{\gamma}{T}\frac{d}{(e-1)T}\int_0^{t_1}\left[t_1t+\frac{t_1^2t}{2T}+\frac{\alpha t_1^{\beta+1}.t}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}t-t^2\right.$$
$$\left.-\frac{t^3}{2T}-\alpha t_1t^{\beta+1}-\frac{\alpha t_1^2t^{\beta+1}}{2T}+\frac{\alpha\beta t^{\beta+2}}{\beta+1}+\frac{\alpha\beta t^{\beta+3}}{2T(\beta+2)}\right]dt$$

$$=\frac{h}{T}\frac{d}{(e-1)T}\left[t_1^2+\frac{t_1^3}{2T}+\frac{\alpha t_1^{\beta+2}}{\beta+1}+\frac{\alpha t_1^{\beta+3}}{T(\beta+2)}-\frac{t_1^2}{2}-\frac{t_1^3}{6T}\right.$$
$$\left.-\frac{\alpha t_1^{\beta+2}}{\beta+2}-\frac{\alpha t_1^{\beta+3}}{2T(\beta+1)}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha\beta t_1^{\beta+3}}{2T(\beta+2)(\beta+3)}\right]$$
$$+\frac{\gamma d}{T(e-1)T}\left[\frac{t_1^3}{2}+\frac{t_1^4}{4T}+\frac{\alpha t_1^{\beta+3}}{2(\beta+1)}+\frac{\alpha t_1^{\beta+4}}{2T(\beta+2)}-\frac{t_1^3}{3}\right.$$
$$\left.-\frac{t_1^4}{8T}-\frac{\alpha t_1^{\beta+3}}{\beta+2}-\frac{\alpha t_1^{\beta+4}}{2T(\beta+2)}+\frac{\alpha\beta t_1^{\beta+3}}{(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2T(\beta+2)(\beta+4)}\right]$$

$$=\frac{h}{T^2}\frac{d}{e-1}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha\beta t_1^{\beta+3}}{2T(\beta+1)(\beta+2)}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right.$$
$$\left.+\frac{\alpha\beta t_1^{\beta+3}}{2T(\beta+2)(\beta+3)}\right]+\frac{y}{T^2}\frac{d}{e-1}\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}-\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+1)(\beta+2)}\right.$$
$$\left.+\frac{\alpha\beta t_1^{\beta+3}}{(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2T(\beta+2)(\beta+4)}\right]$$

$$=\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha\beta t_1^{\beta+3}}{T(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$

$$+\frac{\gamma d}{T^2(e-1)}\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}+\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2T(\beta+2)(\beta+4)}\right]. \quad \ldots(1.12)$$

Total average cost per time is given by

$$K(T_1)=\frac{CD}{T}+\text{Average Holding cost + Average Shortage cost}$$

$$=\frac{CD}{T}+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha\beta t_1^{\beta+3}}{T(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$

$$+\frac{\gamma d}{T^2(e-1)}\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}+\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2T(\beta+1)(\beta+2)}\right]$$

$$+\frac{C_2 d}{T(e-1)}\left(2Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}-eT\right)$$

$$K(T_1)=\frac{cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}-Te^{\frac{t_1}{T}}+T\right]$$

$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha\beta t_1^{\beta+3}}{T(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$

$$+\frac{\gamma d}{T^2(e-1)}\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}+\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2T(\beta+2)(\beta+4)}\right]$$

$$+\frac{c_2 d}{T(e-1)}\left(2Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}-eT\right) \quad \ldots(1.13)$$

The necessary conditions for minimum the total average costs $K(t_1,T)$ are

$$\frac{\partial K}{\partial T}=0 \quad \text{and} \quad \frac{\partial K}{\partial t_1}=0 \quad \ldots(1.14)$$

Now $\frac{\partial K}{\partial T}=0$, gives

$$-2C\left[t_1+\frac{3t_1^2}{4T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{3\alpha t_1^{\beta+2}}{2T(\beta+2)}-\frac{Te^{\frac{t_1}{T}}}{2}-\frac{t_1e^{\frac{t_1}{T}}}{2}+\frac{T}{2}\right]$$

$$-3h\left[\frac{t_1^2}{3}+\frac{t_1^3}{3T}+\frac{\alpha\beta t_1^{\beta+3}}{T(\beta+1)(\beta+3)}+\frac{2\alpha\beta t_1^{\beta+2}}{3(\beta+1)(\beta+2)}\right]$$

$$-3\gamma\left[\frac{2t_1^3}{9}+\frac{t_1^4}{8T}+\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2T(\beta+2)(\beta+4)}\right]$$

$$+C_2 t_1^2 e^{\frac{t_1}{T}} = 0. \qquad \ldots(1.15)$$

and $\dfrac{\partial K}{\partial t_1} = 0$, gives

$$C\left[1+\frac{t_1}{T}+\alpha t_1^{\beta}+\frac{\alpha t_1^{\beta+1}}{T}-e^{\frac{t_1}{T}}\right]$$
$$+h\left[t_1+\frac{t_1^2}{T}+\frac{\alpha\beta t_1^{\beta+1}}{\beta+1}+\frac{\alpha\beta t_1^{\beta+2}}{T(\beta+1)}\right]$$
$$+\gamma\left[\frac{t_1^2}{2}+\frac{t_1^3}{2T}+\frac{\alpha\beta t_1^{\beta+2}}{2(\beta+2)}+\frac{\alpha\beta t_1^{\beta+3}}{2T(\beta+2)}\right]$$
$$+C_2\left(Te^{\frac{t_1}{T}}-\frac{t_1}{T}e^{\frac{t_1}{T}}\right)=0. \qquad \ldots(1.16)$$

Equation (1.15) and (1.16) gives the optimum values of T and t_1 respectively. Provided

$$\frac{\partial^2 K}{\partial T^2}>0, \quad \frac{\partial^2 K}{\partial t_1^2}>0 \quad \text{and} \quad \left(\frac{\partial^2 K}{\partial T^2}\right)\left(\frac{\partial^2 K}{\partial t_1^2}\right)-\left(\frac{\partial^2 K}{\partial T \partial t_1}\right)^2>0 \ldots(1.17)$$

Case I: In case of finite planning horizon, the total average cost is given by

$$K(t_1)=\frac{Cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}-Te^{\frac{t_1}{T}}+T\right]$$
$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha\beta t_1^{\beta+3}}{T(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+2}}{T(\beta+1)(\beta+2)}\right]$$
$$+\frac{\gamma d}{T^2(e-1)}\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}+\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2T(\beta+2)(\beta+4)}\right]$$
$$+\frac{C_2 d}{T(e-1)}\left[2Te^{\frac{t_1}{T}}-t_1 e^{\frac{t_1}{T}}-eT\right] \qquad \ldots(1.18)$$

Sub case 1: When $\beta=1$

Sub case 2: When $\beta=2$

Sub case 3: When $\beta=3$

Case II: When T = 1, the total average cost is given by

$$K(t_1)=\frac{Cd}{e-1}\left[t_1+\frac{t_1^2}{2}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{\beta+2}-e^{t_1}+1\right]$$
$$+\frac{hd}{e-1}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3}+\frac{\alpha\beta t_1^{\beta+3}}{(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$

$$+\frac{\gamma d}{e-1}\left[\frac{t_1^3}{6}+\frac{t_1^4}{8}+\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+4}}{2(\beta+2)(\beta+4)}\right]$$
$$+\frac{C_2 d}{e-1}\left[2e^{t_1}-t_1 e^{t_1}-e\right] \quad \ldots(1.19)$$

Sub case I: When $\beta=1$, then deterioration rate becomes constant in case of finite planning horizon

$$K(t_1)=\frac{cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^2}{2}+\frac{\alpha t_1^3}{3T}-Te^{\frac{t_1}{T}}+T\right]$$
$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha t_1^4}{8T}+\frac{\alpha t_1^3}{6}\right]+\frac{\gamma d}{T^2(e-1)}$$
$$\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}+\frac{\alpha t_1^4}{24}+\frac{\alpha t_1^5}{30T}\right]+\frac{c_2 d}{T(e-1)}\left(2Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}-eT\right).$$
$$\ldots(1.20)$$

Therefore

$$\frac{dK(t_1)}{dt_1}=\frac{cd}{T^2(e-1)}\left[1+\frac{t_1}{T}+\alpha t_1+\frac{\alpha t_1^2}{T}-e^{\frac{t_1}{T}}\right]$$
$$+\frac{hd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{T}+\frac{\alpha t_1^3}{2T}+\frac{\alpha t_1^2}{2}\right]+\frac{\gamma d}{T^2(e-1)}$$
$$\left[\frac{t_1^2}{2}+\frac{t_1^3}{2T}+\frac{\alpha t_1^3}{6}+\frac{\alpha t_1^4}{6T}\right]+\frac{c_2 d}{T^2(e-1)}\left(Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}\right)$$

For minimum total average cost $\frac{dK(t_1)}{dt_1}=0$

$$\Rightarrow \quad C\left(1+\frac{t_1}{T}+\alpha t_1+\frac{\alpha t_1^2}{T}-e^{\frac{t_1}{T}}\right)+h\left(t_1+\frac{t_1^2}{T}+\frac{\alpha t_1^3}{2T}+\frac{\alpha t_1^2}{2}\right)$$
$$+\gamma\left(\frac{t_1^2}{2}+\frac{t_1^3}{2T}+\frac{\alpha t_1^3}{6}+\frac{\alpha t_1^4}{6T}\right)+c_2\left(Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}\right)=0. \ \ldots(1.21)$$

Sub case 2. When $\beta=2$, deterioration rate will become a variable linear function of time, then total average cost becomes

$$K(t_1)=\frac{Cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^3}{3}+\frac{\alpha t_1^4}{4T}-Te^{\frac{t_1}{T}}+T\right]$$
$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{2\alpha t_1^5}{15T}+\frac{\alpha t_1^4}{6}\right]+\frac{\gamma d}{T^2(e-1)}$$

$$\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}+\frac{\alpha t_1^5}{20}+\frac{\alpha t_1^6}{24T}\right]+\frac{c_2 d}{T^2(e-1)}\left(2T^2 e^{\frac{t_1}{T}}-t_1 T e^{\frac{t_1}{T}}-eT^2\right) \quad \ldots(1.22)$$

For minimum value of $K(t_1)$, $\frac{dK(t_1)}{dt_1}=0$

$$\Rightarrow \quad C\left(1+\frac{t_1}{T}+\alpha t_1^2+\frac{\alpha t_1^3}{T}-e^{\frac{t_1}{T}}\right)+h\left(t_1+\frac{t_1^2}{T}+\frac{2\alpha t_1^4}{3T}+\frac{2\alpha t_1^3}{3}\right)$$
$$+\gamma\left(\frac{t_1^2}{2}+\frac{t_1^3}{2T}+\frac{\alpha t_1^4}{4}+\frac{\alpha t_1^5}{4T}\right)+c_2\left(Te^{\frac{t_1}{T}}-t_1 e^{\frac{t_1}{T}}\right)=0. \quad \ldots(1.23)$$

Sub case 3. When $\beta=3$, then deterioration rate will become a quadratic function of time. Then total average cost equation becomes

$$K(t_1)=\frac{Cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^4}{4}+\frac{\alpha t_1^5}{5T}-Te^{\frac{t_1}{T}}+T\right]$$
$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha t_1^6}{8T}+\frac{3\alpha t_1^5}{20}\right]+\frac{\gamma d}{T^2(e-1)}$$
$$\left[\frac{t_1^3}{6}+\frac{t_1^4}{8T}+\frac{\alpha t_1^6}{20}+\frac{3\alpha t_1^7}{70T}\right]+\frac{c_2 d}{T(e-1)}\left[2Te^{\frac{t_1}{T}}-t_1 e^{\frac{t_1}{T}}-eT\right]. \quad \ldots(1.24)$$

For minimum value of $K(t_1)$, $\frac{dK(t_1)}{dt_1}=0$

$$\Rightarrow \quad C\left(1+\frac{t_1}{T}+\frac{\alpha t_1^3}{1}+\frac{\alpha t_1^4}{T}-e^{\frac{t_1}{T}}\right)+h\left(t_1+\frac{t_1^2}{T}+\frac{3\alpha t_1^5}{4T}+\frac{3\alpha t_1^4}{4}\right)$$
$$+\gamma\left(\frac{t_1^2}{2}+\frac{t_1^3}{2T}+\frac{3\alpha t_1^5}{10}+\frac{3\alpha t_1^6}{10T}\right)+c_2\left(Te^{\frac{t_1}{T}}-t_1 e^{\frac{t_1}{T}}\right)=0. \quad \ldots(1.25)$$

For minimum value of $K(t_1)$ $\frac{\partial K}{\partial t_1}=0$, which gives

$$C\left[1+t_1+\alpha t_1^{\beta}+\alpha t_1^{\beta+1}-e^{t_1}\right]+h\left[t_1+t_1^2+\frac{\alpha\beta t_1^{\beta+1}}{\beta+1}+\frac{\alpha\beta t_1^{\beta+1}}{\beta+1}\right]$$
$$+\gamma\left[\frac{t_1^2}{2}+\frac{t_1^3}{2}+\frac{\alpha\beta t_1^{\beta+2}}{2(\beta+2)}+\frac{\alpha\beta t_1^{\beta+3}}{2(\beta+2)}\right]$$
$$+C_2\left(Te^{t_1}-t_1 e^{t_1}\right)=0. \quad \ldots(1.26)$$

Case III. If $\gamma = 0$, then holding cost will becomes constant. Total averages cost becomes

$$K(t_1)=\frac{cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^{\beta+1}}{\beta+1}+\frac{\alpha t_1^{\beta+2}}{T(\beta+2)}-Te^{\frac{t_1}{T}}+T\right]$$
$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha\beta t_1^{\beta+3}}{T(\beta+1)(\beta+3)}+\frac{\alpha\beta t_1^{\beta+2}}{(\beta+1)(\beta+2)}\right]$$
$$+\frac{c_2 d}{T(e-1)}\left(2Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}-eT\right). \quad \ldots(1.27)$$

For minimum total averages cost, $\frac{dK(t_1)}{dt_1}=0$

$$\Rightarrow \quad C\left(1+\frac{t_1}{T}+\alpha t_1^{\beta}+\frac{\alpha t_1^{\beta+1}}{T}-e^{\frac{t_1}{T}}\right)+h\left(t_1+\frac{t_1^2}{T}+\frac{\alpha\beta t_1^{\beta+2}}{T(\beta+1)}+\frac{\alpha\beta t_1^{\beta+1}}{(\beta+1)}\right)$$
$$+c_2\left(Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}\right)=0. \quad \ldots(1.28)$$

Sub case 1. When $\beta=1$, then deterioration rate will become constant.

$$K(t_1)=\frac{Cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^2}{2}+\frac{\alpha t_1^3}{3T}-Te^{\frac{t_1}{T}}+T\right]$$
$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha t_1^4}{8T}+\frac{\alpha t_1^3}{6}\right]$$
$$+\frac{C_2 d}{T(e-1)}\left[2Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}-eT\right]. \quad \ldots(1.29)$$

For minimum value of $K(t_1)$, $\frac{dK(t_1)}{dt_1}=0$

$$\Rightarrow \quad C\left(1+\frac{t_1}{T}+\alpha t_1+\frac{\alpha t_1^2}{T}-e^{\frac{t_1}{T}}\right)+h\left(t_1+\frac{t_1^2}{T}+\frac{\alpha t_1^3}{2T}+\frac{\alpha t_1^2}{2}\right)$$
$$+C_2\left(Te^{\frac{t_1}{T}}-t_1e^{\frac{t_1}{T}}\right)=0. \quad \ldots(1.30)$$

Sub case 2. When $\beta=2$, then deterioration rate will become a variable linear function of time.

$$K(t_1)=\frac{Cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^3}{3}+\frac{\alpha t_1^4}{4T}-Te^{\frac{t_1}{T}}+T\right]$$
$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{2\alpha t_1^5}{15T}+\frac{\alpha t_1^4}{6}\right]$$

$$+\frac{C_2 d}{T(e-1)}\left[2T^2 e^{\frac{t_1}{T}} - t_1 T e^{\frac{t_1}{T}} - eT^2\right]. \quad \ldots(1.31)$$

For minimum value of $K(t_1)$, $\frac{dK(t_1)}{dt_1}=0$

$$\Rightarrow \quad C\left(1+\frac{t_1}{T}+\alpha t_1^2+\frac{\alpha t_1^3}{T}-e^{\frac{t_1}{T}}\right)+h\left(t_1+\frac{t_1^2}{T}+\frac{2\alpha t_1^4}{3T}+\frac{2\alpha t_1^3}{3}\right)$$
$$+C_2\left(Te^{\frac{t_1}{T}}-t_1 e^{\frac{t_1}{T}}\right)=0. \quad \ldots(1.32)$$

Sub case 3. When $\beta=3$, then deterioration rate will become a quadratic function of time. Then total average cost

$$K(t_1)=\frac{Cd}{T^2(e-1)}\left[t_1+\frac{t_1^2}{2T}+\frac{\alpha t_1^4}{4}+\frac{\alpha t_1^5}{5T}-Te^{\frac{t_1}{T}}+T\right]$$
$$+\frac{hd}{T^2(e-1)}\left[\frac{t_1^2}{2}+\frac{t_1^3}{3T}+\frac{\alpha t_1^6}{8T}+\frac{3\alpha t_1^5}{20}\right]$$
$$+\frac{C_2 d}{T(e-1)}\left[2Te^{\frac{t_1}{T}}-t_1 Te^{\frac{t_1}{T}}-eT\right]. \quad \ldots(1.33)$$

For minimum total average cost, $\frac{dK(t_1)}{dt_1}=0$,

$$\Rightarrow \quad C\left(1+\frac{t_1}{T}+\alpha t_1^3+\frac{\alpha t_1^4}{T}-e^{\frac{t_1}{T}}\right)+h\left(t_1+\frac{t_1^2}{T}+\frac{3\alpha t_1^5}{4T}+\frac{3\alpha t_1^4}{4}\right)$$
$$+C_2\left(Te^{\frac{t_1}{T}}-t_1 e^{\frac{t_1}{T}}\right)=0. \quad \ldots(1.34)$$

CONCLUSION

We have endeavored to develop a warehouse inventory system with a very realistic and practical deterioration rate. The effect of deterioration of physical goods in stock is very realistic feature of inventory control. In this model deterioration rate at any item is assumed to follow parameter Weibull distribution function of time. This deterioration rate is suitable for items with and without life-period. The warehouse inventory problem is an intriguing yet practical topic of decision science. The warehouse model can be applied to many practical situations, due to introduction of open market policy; the business competition becomes very high to occupy maximum possible market. As a result, the management of the departmental store is bounded to hire a separate warehouse on rental basis at a distance place for storing of excess items.

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